Accounting for Mathematicians

by OSOE Project

▼ Details

This visual guide is part of a collection of documents created by the One Student One ERP (OSOE) project in collaboration with Institut Mines Telecom, Telecom Bretagne, Dresden University of Technology and the South Westfalia University of Applied Sciences. It can be used to teach modern ERP theory and practice to undergraduate students or professionals.

Agenda

- Account Space
- Legislation Space
- Currency Space & Exchange
- Transactions and Journals
- Invariants
- Reports
- Profits

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The goal of this lesson is to teach accounting in a way which will be valid in the next 100 years or more. For that purpose, we introduce in the lesson the concepts of accounting using a set theory formalism.

We believe that too much fashion, too many fuzzy words are used in the current approaches of accounting education. Students learn about the so-called general accounting, which is not general at all and which is usually presented in a way which is specific to each legislation. Students learn about the so-called cost accounting, which has no relation to costs. Students learn about analytical accounting, with a singular, instead of learning about accounting analytics, with a plural. Students learn about activity based accounting, which is just one example of accounting analytics.

We hope, through this lesson, to exhibit the concept of universal accounting in such a way that students can learn a model which is useful to understand and explain all the types of accounting which exist or will be created in the near or distant future, based on the ever evolving fashion in corporate management.

The Account Space: A

$$A = /a_1 \times /a_2 \times \ldots \times /a_n$$

$/a_1$	General Accounts
$/a_2$	Parties (clients, supplies, business units, etc.)
$/a_3$	Project
$/a_4$	Activities

▼ Details

The first concept in accounting is the `Account`. An account is a point in an arbitrary N-dimensional discrete space. Some people like to give names to those dimensions, names which will likely become irrelevant in a century. Dimension 1 is called "General Account". It represents the economic nature of the account: income, expense, inventory, etc. Dimension 2, the "Party", represents a supplier, a client, a business unit or any 3rd party. Dimension 3, the "Project", represents a project in a company which does project management. Dimension 4, the “Activity”, represents activities such as gardening. More dimensions could be used to provide more powerful accounting analytics.

Legislation: L

$$L: A \longrightarrow \mathbb{N}$$ General Mapping

$$(a_1, a_2, a_3, \ldots, a_n) \longrightarrow L_1\ (a_1)$$ Simplified mapping

▼ Details

Governments, especially in countries of Civil Law, like to define an official space of accounts, called **L**. This space of account contains usually a few thousand standard account names which are required to be used to produce legislation dependent tax reports and exchange data.

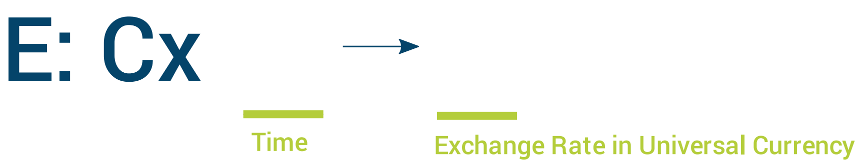
This space of account can evolve. Many accountants believe that Dimension 1 of the space of accounts is the space of accounts provided by the government. This creates a lot of troubles each time the legislation changes or whenever a company has subsidiaries in different countries.

To solve all these problems, we define here a Legislation L as a mapping between the space of Accounts A and a single dimension discrete space which represents one possible legislation. Thanks to this approach, called indirection, our space of accounts no longer depends on political fashion, as long as A is a superset of L.

In countries, especially in countries of Common Law, governments prefer to define accounting principles through quite ambiguous rules. Those rules are then interpreted by chartered accountants who use their legal expertise to create their own space of account. It is like the Legislation L is provided here by chartered accountants instead of being provided by the government.

The Currency Space: C

$$C = \{EUR, USD, JPY, XOF, \ldots\}$$



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Accounting lives in a world of currencies. We introduce a space of currencies: **C**. C is a single dimension discrete space. **A** currency exchange function E is also introduced, whether currencies can be really exchanged or not. The currency exchange space is required because most tax reports must be produced using a single currency, the currency of the country, although most companies use multiple currencies.

The currency exchange function **E** provides for a given currency c in **C** an exchange rate expressed in a universal currency.

Note: this currency exchange model is a simplified symmetrical model for currency exchange. Reality is more complex and is based on asymmetric currency to currency exchange rates. However, introducing a more complex model for currency exchange would not help for the education purpose of this lesson.

Accounting Transactions: T

$$T : \_ \rightarrow Ax\ xAx\ xCx$$

Amount in Price Currency

Destination Amount

Source Amount

Transaction Line Number

▼ Details

An accounting transaction is a finite series of sextuples. Each item is a finite series is called a transaction line.

Each sextuple is made of 3 couples The first couple represents the account of the transaction at the `source` and the amount in the currency of the source. The second couple represents the account of the transaction at the `destination` and the amount of the transaction in the currency of the destination. The third tuple represents the `currency` and the amount in that currency of the transaction.

Let us give an example to explain. If the source represents the supplier and if the destination represents the client. If the supplier sells a product to a client with a price expressed in USD. If the supplier is a company in Germany which operates in EUR and the client is a company in Japan which operates in JPY, the amount of the invoice is expressed in USD. The supplier registers the invoice in its accounting using for example an “Income” or a “Receivable” account and expresses the amount in EUR. The client registers the invoice in its accounting using for example an “Expense” or a “Payable” account and expresses the amount in JPY.

We have just described the simplest universal model of an accounting transaction. In our example, the accounting transaction has 2 lines. One line with Income at the source and Expense at the destination. One line with Receivable at the source and Payable at the destination.

Transaction Journal: J

$$J : \quad \rightarrow x(\quad \rightarrow Ax\ xAx\ xCx\ )$$

$$J\ (n)=(t,T)$$

▼ Details

Account Transactions form a finite series called the Journal `J`. Each item of the Journal defines a date `t` and a transaction `T`.

Debit & Credit Invariance: a Myth

$$T(i)=(a_i,\ s_i,\ a'_i,\ d_i,\ c_i,\ p_i)$$

$$\sum_{i\in} s_i=0 \qquad \sum_{i\in} d_i=0$$

Tax Debit = Tax Credit

$$\sum_{i\in} E(c_i,t).p_i=0$$

Price Debit = Price Credit

▼ Details

Every transaction in traditional accounting should meet the "debit and credit" invariant principle. Technically, it is quite simple. The total of amounts at the source is zero. The total of amounts at the destination is zero. The total of trade amounts converted in universal currency is zero.

However, in "real world accounting", accountants call a positive value a debit. They call a negative value a credit. This way, they do not need to display any negative signs in their reports.

Assets, a positive value, are displayed as a debit. Debts, a negative value, are displayed as credit.

Moreover, they consider that sales income are future profits, that profits are debts to the owner of the company, and that income should thus be considered as a negative value, just like debts. Just the same way, expenses are future losses, which will reduce the capital of the company, i.e. reduce a debt to the owner of the company, and should thus be considered positive values. It is a bit weird at first but consistent.

Income, a negative value, is displayed as credit. Expense, a positive value, is displayed as debit.

Now, where does this debit and credit invariant comes from? The sad truth is that this debit and credit invariant has no well defined rationale. Many companies in the world do accounting without following this invariant, even if accountants in countries of Civil Law hate to hear that.

The only rationale of the debit and credit invariant is to help making sure that every transaction can be partitioned in 2 subsets of opposite total, consistently with the partitioning of accounts introduced later. However, it is not the only way to reach that goal.

Supplier & Client Invariant

$$p_i.E(c_i,t) = s_i.E(c_{source},t)$$

$$p_i.E(c_i,t) = -d_i.E(c_{destination},t)$$

$$c_{source} = Currency(a_{i,2})$$

$$c_{destination} = Currency(a'_{i,2})$$

▼ Details

The supplier and client invariant is a real and universal invariant which most accountants are not aware of. It relates to the principles of chemistry "nothing is created, nothing disappears, everything is transformed".

What this invariant says is that whenever a supplier has an income, its client has an expense of equivalent amount, consistently with the exchange rate of the transaction currency.

This invariant applies to many situations, even in situations where the same company acts as supplier and as client. For example, amortization of an asset, could be represented with a single transaction line with same company in Dimension 2 of the space of accounts on both sides. The company at the source acts as a supplier and provides a share of machine which reduces the Immobilisation amount (an asset). The company at the destination acts as a client which consumes a share of machine, just like it would rent the machine, which is represented as an Expense. No other line is required since no cash is transferred and nothing would anyway appear at consolidation time. Another common example is the case of currency transfer, which is represented much more efficiently with a single transaction with same company at both ends. Last but not least, some companies prefer to enter only a single for every sales they make and then, at the end of the month, gather all sales transactions into a single receivable.

In short: the invariance of Debit & Credit is a myth, the supplier and client invariance is universal.

Tax Report

$$\sum_{T \in J} \left[\sum_{\substack{a_i \in R \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} s_i - \sum_{\substack{a'_i \in R \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} d_i \right]$$

Where

$$R \subset A$$

▼ Details

Tax reports are made by calculating the total of transaction amounts expressed in the national currency of a single organisation for a subset of the space of accounts. Both source and destination amounts should be taken into account since the organisation for which the tax report is calculated could appear on both side.

The report space **R** is a subset of the space of accounts A.

Consolidated Tax Report

$$\sum_{T \in J} \left[\sum_{\substack{a_i \in R \wedge a'_i \notin R \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} s_i - \sum_{\substack{a'_i \in R \wedge a_i \notin R \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} d_i \right]$$

Where

$$R \subset A$$

▼ Details

Consolidated Tax reports are made by calculating the total of transaction amounts expressed in the national currency of a single organisation for a subset of the space of accounts, as long one side of the transaction has an account which is not part of the report space R.

Auxiliary Tax Report

$$\sum_{T \in J} \left[\sum_{\substack{a_i \in R_1 \wedge a'_i \in R_2 \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} s_i - \sum_{\substack{a'_i \in R_1 \wedge a_i \in R_2 \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} d_i \right]$$

Where

$$R_1 \subset A \wedge R_2 \subset A$$

▼ Details

Consolidated Tax reports are made by calculating the total of transaction amounts expressed in the national currency of a single organisation for two subsets of the space of accounts : `R1 and R2`.

R1 represents all accounts (ex. payable) of a given organisation A. R2 represents all accounts (ex. Receivable) of another organisation B. The auxiliary report will produce in this example the amount of payable from B to A which has been accounted by A as receivable and by B as payable.

Weighted Tax Report

$$\sum_{T \in J} \sum_{(a_i, s_i, a'_i, d_i, c_i, p_i) \in T} \sum_{a \in R} \left[W(a_i,a).s_i - W(a'_i,a).d_i \right]$$

Where

$$W : A \times A \longrightarrow$$

$$R \subset A$$

▼ Details

Weighted Tax reports consists of using a weight function W instead of using mere set membership, which is just a simple case of weight with values in 0 and 1.

Trade Report in €: R

$$\sum_{T \in J} \left[\sum_{\substack{a_i \in R \wedge c_i = EUR \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} p_i - \sum_{\substack{a'_i \in R \wedge c_i = EUR \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} p_i \right]$$

Where

$$R \subset A$$

▼ Details

Trade reports consist of making reports using a given currency and using the trade amount of the transaction.

Legislation Report

$$\sum_{T \in J} \left[\sum_{(a_i,\, s_i,\, a'_i,\, d_i,\, c_i,\, p_i) \in T}^{L(a_i) \in R} s_i - \sum_{(a_i,\, s_i,\, a'_i,\, d_i,\, c_i,\, p_i) \in T}^{L(a'_i) \in R} d_i \right]$$

Where

$$R \subset //$$

▼ Details

Legislation reports consist are tax reports which define the subset of accounts on which the total should be made through membership tests on the Legislation function L.

An Example of Account Partition

$$A = A_{acme} \quad (A \setminus A_{acme})$$

$$A_{acme} = \mathit{Asset} \quad \mathit{Liability} \quad \mathit{Income} \text{ U } \mathit{Expense}$$

▼ Details

Accountants like to create partitions.

First, they make a difference between accounts of “our” company, called here Aacme and other accounts. Inside Aacme, they make a difference between accounts called Asset, Liability, Income and Expense. Those accounts represent traditional ways of creating accounts based on legal principles, which are rather universal. Yet, in some countries, Capital is considered as a 5th partition item whereas in others Capital is part of Liabilities. This is just a matter of Habit and Law.

Other ways of partitioning could be introduces of course.

Profit

$$\sum_{T \in J} \left[\sum_{(a_i,\, s_i,\, a'_i,\, d_i,\, c_i,\, p_i) \in T}^{a_i \in \text{Income U Expense}} s_i - \sum_{(a_i,\, s_i,\, a'_i,\, d_i,\, c_i,\, p_i) \in T}^{a'_i \in \text{Income U Expense}} d_i \right]$$

$$= \mathit{profit}$$

▼ Details

Accounts then define words, based on the partitioning. Profit is for example of the sum of transactions lines amounts for accounts which are either Income or Expense. Remember here that Expense is a negative value and Income a positive value, since we do not use Debit or Credit.

Asset

$$\sum_{T \in J} \left[\sum_{\substack{a_i \in Asset \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} s_i - \sum_{\substack{a'_i \in Asset \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} d_i \right] = asset$$

▼ Details

Assets are defined by summing transaction amounts which account is in Asset.

Liability

$$\sum_{T \in J} \left[\sum_{\substack{a_i \in Liability \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} s_i - \sum_{\substack{a'_i \in Liability \\ (a_i, s_i, a'_i, d_i, c_i, p_i) \in T}} d_i \right] = liability$$

▼ Details

The same goes for Liability.

Asset and Liability Invariance

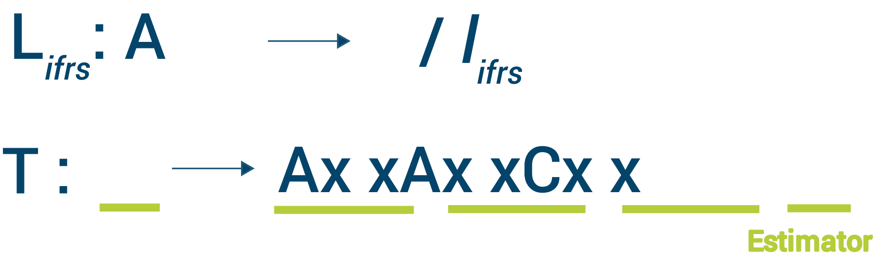
$$asset + liability + profit = 0$$

▼ Details

And, by magic, general principles such as : asset + liability + profit = 0 can be exhibited. This sounds like magic, but it just means that by summing transactions which lines implement the invariance of Debit and Credit, and by making a partition of the account of space, the total is obviously always zero.

Things would of course be different if the invariance of Debit and Credit was not respected. And this is probably where the rationale of this invariance comes from. By partitioning the space of accounts, certain accounting principles can be introduced. For example, it is usual to consider that every Income (ex. sales) must have an equal counterpart in the form of an Asset (this is of course simplified). Every Expense must have an equal counterpart in the form of Liability. Whenever Debit & Credit invariance is not respected at the level of a transaction, it is certain that such principles can not be fulfilled. Debit & Credit invariance is thus only a simple way, a kind of hack, to help accountants reduce input mistakes which would else break general reporting principles related to the partitioning of the space of Accounts.

IFRS Accounting



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IFRS is a tentative of creating international accounting standards in the 21st century. It will likely fail because it is based on fuzzy principles which rely on the interpretation of chartered accounts and which are sensitive to fashion. In practice, it is yet another legislation, which can call here Lifrs.

However, one key principle of IFRS, the economic value, is interesting in our model. IFRS requires accountant to consider the economic value of every asset. This means that whenever a transaction is recorded for a given receivable asset, even if its formal value is 100, it should be recorded by taking into account its real value, i.e. the formal value multiplied by the estimated probability of payment.

This would be a major change in the accounting model, and possibly a useful change. Either by adding new items to the transactions in order to categorize transaction lines, or by adding new dimensions to transaction lines and hold various estimators, it is possible to extend the universal model of accounting to support fine grained evaluation of the economic value of each transaction. Combined with data mining to build estimates from transaction history, this would be a way to introduce more science in the work of chartered accounts for who, IFRS, is mostly nowadays a way to add unpredictable transactions with little rationale to the formal input of the accountant and maximize the assets of a company for the great benefit of the stockholder.